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A new conflict management method in Dempster–Shafer theory

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Abstract
Conflict management is a hot research topic in Dempster–Shafer theory which is used to avoid the counterintuition problem of combination results. In the present conflict management methods, conflicting evidence is assigned to smaller weight to reduce its influence on the combination result. However, these methods will be disabled when similarity collision occurs. In this article, a new conflict management method based on similarity and Basic Probability Assignment-Matrix is proposed; in the proposed method, similarity collision is diminished by computing the index of each element in evidence, and then, a more reasonable evidence weight can be determined in this way. In the end, two experiments are set to compare the results of several different methods, and the results illustrate that the conflicting evidence will be assigned to a smaller weight in our method than in others.

Keywords
Dempster–Shafer theory, similarity of evidence, conflict management, evidence conflict, basic probability assignment

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Introduction

Dempster–Shafer (D-S) theory[1] is an effective tool to make a decision from several answers with ambiguity. To each answer, the probability that it must be true is denoted as *Bel* and the probability that it cannot be false is denoted as *Pl*. *Bel* is also marked as BPA (basic probability assignment) or mass function m. All the BPAs of a same question will constitute a D-S evidence. A set of evidence can be combined into a new piece of evidence by combination rules, and the largest BPA in the combination result represents the decision. Traditionally, the biggest BPA in BOE (Body of Evidence) is the answer supported by the evidence. Based on the attributes above, evidence theory is widely applied in reliable analysis,[2–5] relationship computing,[6] making decision,[7–10] and optimal problems.[11–13]

However, the evidence we gathered may be not accurate and even the answer supported by each evidence may be different.[14] This phenomenon is the conflict of evidence,[15,16] and the defects of combining conflicting evidence directly mainly lies on:[17]

1. Combination a set of conflicting evidence may assign 100% probability to a BPA which is counterintuitive.
2. The combination result may be totally wrong due to a highly conflicting evidence.
3. The BPA equal to zero will always be zero in combination whatever the other BOES.

To overcome the defects above, many scholars proposed their conflict management methods to control the influence of conflict evidence by assigning weight to each evidence. Based on evidence similarity values, weight of each evidence is determined. Wen et al.[18]

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proposed a similarity calculation method based on computing the cosine value between evidence. Zhao et al.[6] proposed another conflict management method based on computing the intersection part of evidence. In addition, similarity of evidence can be obtained by computing the distance of evidence.

Cuzzlion[19] explained the meaning of evidence distance in geometric view which stands for the process that transforms the difference between evidence into the distance of points in geometric space. Jousselme et al.[20] proposed an evidence distance calculation method based on the difference of BOEs and the number of elements in BPA. Sunberg and Rogers[21] modified Jousselme's distance by comparing the difference of the biggest and the smallest BPA in evidence.

Based on similarity of evidence, Deng et al.[22] proposed a conflict management method by transferring similarity values into the weight of evidence. Wang et al.[23] modified Murphy's combination rule by computing evidence distance. Chin and Fu[24] proposed another conflict management method based on evidence distance and mass function. Su et al.[25] proposed a combination method for determining dependent evidence based on conflict management. Jiang et al.[26] proposed a fault diagnosis method based on managing conflicting evidence. Yang and Han[27] proposed an uncertainty measure method based on conflict management. Sankararaman and Mahadevan[28] proposed a validation measure method based on conflict management.

But similarity calculation methods used by many scholars are not as faultless as they believe. Similarity calculation is the key in conflict management, and the difference of BOEs should be represented by the difference of similarity values. However, the phenomenon that two different pairs of evidence share a same similarity value is easy to be found, and this phenomenon is the collision of similarity which is mainly caused by the BPA sequence in BOE is not computed, and the exchange of BPA sequence may not change the similarity calculation result.

In this article, a matrix named BPA-Matrix (Basic Probability Assignment-Matrix) that stands for BPA sequence in BOE is introduced. With the introduction of BPA-Matrix, collision of similarity is diminished and weight of evidence becomes more reasonable. The frame of this article is as follows: section "Introduction" introduces D-S theory and the necessity of conflict management. Section "Preliminaries" depicts some important definitions and equations used in conflict management method proposed in this article, and a brief example is given to illustrate the defects in existing conflict management. Section "The method proposed" illustrates the process of the proposed conflict management, and an example is given to illustrate the computing process of BPA-Matrix. Section "Experiments" is the experiment part, and the weight of each evidence determined by several conflict management methods is compared under two different sets of evidence.

Preliminaries

D-S evidence theory

Let Ω be a set containing several exclusive and exhaustive elements: $\{H_1, H_2, H_3, \ldots, H_N\}$, Ω is denoted as the frame of discernment and $P(\Omega)$ is a power set where $P(\Omega) = 2^{\Omega}$. For any subset of $P(\Omega)$ which is marked as A, $Bel(A)$ is the probability that A is true. $Bel(A)$ is also marked as $m(A)$ or BPA, and many BPAs can constitute a BOE

$$m : (m(A), m(B), m(C), \ldots, m(AB), \ldots, m(\Omega), m(\varnothing))$$

In the equation above, A, B, and C are single-element focal elements, and AB is multi-element focal element. $m(A)$ or $m(B)$ is BPA of A or B, and $m(\Omega)$ is the probability that all the focal elements are uncertain. All the BPAs will constitute a piece of D-S theory evidence which is marked as m. A set of evidence can be combined into a new piece of evidence by combination rule, and the combination rule proposed by Dempster is as follows

$$m(A) = \begin{cases} 0, A = \varnothing \\ \dfrac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{C}, A \neq \varnothing \end{cases} \tag{1}$$

$$C = 1 - \sum_{A_i \cap B_j = \varnothing} m_1(A_i) m_2(B_j) \text{ or } C = \sum_{A_i \cap B_j \neq \varnothing} m_1(A_i) m_2(B_j) \tag{2}$$

In equations (1) and (2), m is the combination result of m_1 and m_2. Equations (1) and (2) denote the combination rule when two pieces of evidence are combined, and equations (1) and (2) will be transferred into equations (3) and (4) when the number of evidence is larger than two

$$m(A) = \begin{cases} 0, A = \varnothing \\ \dfrac{\sum_{\cap A_i = A} \prod_{1 \leq i \leq n} m_i(A_i)}{C}, A \neq \varnothing \end{cases} \tag{3}$$

$$C = 1 - \sum_{\cap A_i = \varnothing} \prod_{1 \leq i \leq n} m_i(A_i) \text{ or } C = \sum \prod_{1 \leq i \leq n} m_i(A_i) \tag{4}$$

According to equations (1)–(4), combination rule proposed by Dempster meets the law of commutation and the law of association.[7] Even though evidence can be combined based on combination rule proposed by

Dempster, the combination result may be counterintuitive. A brief example is given in the following section.

Example 1. Assuming m_1 and m_2 are two pieces of evidence under a same frame of discernment, and the BOEs of m_1 and m_2 are given as follows

$$m_1 : m_1(X) = 0.99,\ m_1(Y) = 0.01,\ m_1(Z) = 0.00$$
$$m_2 : m_2(X) = 0.00,\ m_2(Y) = 0.01,\ m_2(Z) = 0.99$$

From equation (2), C is obtained as follows: $C = \sum_{A_i \cap B_j \neq \varnothing} m_1(A_i)m_2(B_j) = 0.0001$.

And, focal elements in combination result can be obtained by equation (1)

$$m(X) = \frac{\sum_{A_i \cap B_j = X} m_1(A_i)m_2(B_j)}{C} = \frac{m_1(X)m_2(X)}{C} = 0.00$$
$$m(Y) = \frac{\sum_{A_i \cap B_j = Y} m_1(A_i)m_2(B_j)}{C} = \frac{m_1(Y)m_2(Y)}{C} = 1.00$$
$$m(Z) = \frac{\sum_{A_i \cap B_j = Z} m_1(A_i)m_2(B_j)}{C} = \frac{m_1(Z)m_2(Z)}{C} = 0.00$$

The combination result of m_1 and m_2 is $m : m(X) = 0.00,\ m(Y) = 1.00,\ m(Z) = 0.00$.The focal element supported in evidence m_1 is X, and the focal element supported in evidence m_2 is Z, but the focal element supported in the combination result is Y, which is counterintuitive. To overcome the shortage above, conflict management of evidence is proposed by many scholars to avoid a counterintuition combination result by assigning weight to each evidence, and the weight of conflicting evidence is smaller than the weight of un-conflicting evidence. The weight is obtained by similarity calculation, and the more similar the evidence is toward the others, the less conflict it causes.

Similarity of evidence

Similarity calculation. As described in section "Introduction," the difference between evidence can be transferred into a value by similarity calculation. And, similarity calculation method proposed by Wen et al.[18] is defined in Definition 1.

Definition 1 (evidence similarity proposed by Wen et al.). Assuming that m_1 and m_2 are two pieces of evidence under a same frame of discernment, the similarity value between m_1 and m_2 is as follows

$$sim_{wen}(m_1, m_2) = \frac{m_1 \cdot m_2^T}{||m_1|| \cdot ||m_2||} \tag{5}$$

$||m_1||$ and $||m_2||$ are the norms of m_1 and m_2, and $m_1 \cdot m_2^T$ is the Cartesian product of m_1 and m_2. Similarity calculation method proposed by Wen et al. represents a series of methods which compute similarity directly. In addition, some scholars proposed their similarity calculation methods based on evidence distance. Jousselme et al.[20] proposed four limits which most evidence distance calculation method should follow:

1. Nonnegativity: $d(m_1, m_2) \geq 0$;
2. Nondegeneracy: $d(m_1, m_2) = 0 \Leftrightarrow m_1 = m_2$;
3. Symmetry: $d(m_1, m_2) = d(m_2, m_1)$;
4. Triangle: $d(m_1, m_2) \leq d(m_1, m_3) + d(m_3, m_2)$.

$d(m_1, m_2)$ is the distance of evidence m_1 and m_2. Based on the limits above, many scholars proposed their evidence distance calculation methods and method proposed by Jousselme is defined in the following section.

Definition 2 (evidence distance proposed by Jousselme et al.). Assuming that m_1 and m_2 are two pieces of evidence under a same frame of discernment, the distance between m_1 and m_2 is as follows

$$d_{i,j} = \sqrt{\frac{1}{2}\left(\vec{m}_i - \vec{m}_j\right)^T D\left(\vec{m}_i - \vec{m}_j\right)} \tag{6}$$

$\vec{m}_i$ and $\vec{m}_j$ are the vector form of m_i and m_j, D is a matrix defined as $D(A, B) = |A \cap B| / |A \cup B|$ where A and B are focal elements. Based on the distance of evidence, similarity of evidence can be obtained as follows

$$sim_{Jou}(m_i, m_j) = 1 - d_{i,j} \tag{7}$$

Collision of similarity. Similarity calculation is a key process in conflict management, but collision of similarity may occur in both the methods proposed by Jousselme et al. and Wen et al. A brief example is given in the following section.

Example 2 (similarity collision). Assuming that m_1, m_2, and m_3 are three pieces of evidence under a same frame of discernment, the BOE of each evidence is shown as follows

$$m_1 : m_1(A) = 0.3,\ m_1(B) = 0.2,\ m_1(C) = 0.1,\ m_1(AC) = 0.4$$
$$m_2 : m_2(A) = 0.1,\ m_2(B) = 0.2,\ m_2(C) = 0.3,\ m_2(AC) = 0.4$$
$$m_3 : m_3(A) = 0.2,\ m(B)_3 = 0.2,\ m(C)_3 = 0.2,\ m_3(AC) = 0.4$$

Based on equation (5), similarity value between m_1 and m_3 in Wen et al.'s method is as follows

$$sim_{wen}(m_1, m_3) = \frac{m_1 \cdot m_3^T}{||m_1|| \cdot ||m_3||}$$
$$= \frac{(0.3, 0.2, 0.1, 0.4) \cdot (0.2, 0.2, 0.2, 0.4)^T}{||(0.3, 0.2, 0.1, 0.4)|| \cdot ||0.2, 0.2, 0.2, 0.4||} = \frac{0.28}{0.29} = 0.97$$

And, similarity value between m_2 and m_3 in Wen et al.'s method is as follows

$$sim_{wen}(m_2, m_3) = \frac{m_2 \cdot m_3^T}{||m_2|| \cdot ||m_3||}$$
$$= \frac{(0.1, 0.2, 0.3, 0.4) \cdot (0.2, 0.2, 0.2, 0.4)^T}{||(0.1, 0.2, 0.3, 0.4)|| \cdot ||0.2, 0.2, 0.2, 0.4||} = \frac{0.28}{0.29} = 0.97$$

According to equation (6), the distance of m_1 and m_3 is as follows

$$d_{1,3} = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_3)^T D(\vec{m}_1 - \vec{m}_3)} =$$
$$\sqrt{\frac{1}{2}(0.1, 0, -0.1, 0)^T \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{bmatrix} (0.1, 0, -0.1, 0)} = 0.1$$

From equation (7), similarity value between m_1 and m_3 based on distance is as follows

$$sim_{Jou}(m_1, m_3) = 1 - d_{1,3} = 1 - 0.1 = 0.9$$

And, distance of m_2 and m_3 is as follows

$$d_{2,3} = \sqrt{\frac{1}{2}(\vec{m}_2 - \vec{m}_3)^T D(\vec{m}_2 - \vec{m}_3)} =$$
$$\sqrt{\frac{1}{2}(-0.1, 0, 0.1, 0)^T \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{bmatrix} (-0.1, 0, 0.1, 0)} = 0.1$$

And, similarity between m_2 and m_3 based on distance is as follows

$$sim_{Jou}(m_2, m_3) = 1 - d_{2,3} = 1 - 0.1 = 0.9$$

According to similarity calculation results above, $sim_{wen}(m_1,m_3) = sim_{wen}(m_2,m_3)$ and $sim_{Jou}(m_1,m_3) = sim_{Jou}(m_2,m_3)$ but $m_1 \neq m_2$. The difference between m_2 and m_3 lies on the exchange of A and C. Since BPA sequence in BOE is not computed in similarity calculation, the alternation of BPA sequence may not change the similarity calculation result. Similarity of evidence is the base of conflict management, and the collision of similarity may lead to the deviation of the weight assigned to evidence. Before introducing the conflict management method in this article, further steps in previous conflict management only based on similarity are shown in section "Conflict management only based on similarity."

Conflict management only based on similarity

Definition 3 (support of evidence). Assuming $m_1, m_2, m_3, \ldots, m_n$ is a set of evidence under a same frame of discernment, and $sim(m_i, m_1), sim(m_i, m_2), sim(m_i, m_3), \ldots, sim(m_i, m_n)$ are similarity values among $m_1, m_2, m_3, \ldots, m_n$, the support value Sup_i of m_i is as follows

$$Sup_i = \sum_{j=1, j \neq i}^{n} sim(m_i, m_j) \tag{8}$$

Support of evidence can be obtained by equation (8), but the range of evidence support is not [0, 1]. Note that support of evidence is different with the focal element supported by a piece of evidence. To realize the weight determination of each evidence, a new parameter named evidence credit is defined in the following section.

Definition 4 (credit of evidence). Assuming that $m_1, m_2, m_3, \ldots, m_n$ is a set of evidence under a same frame of discernment, and $Sup_1, Sup_2, Sup_3, \ldots, Sup_n$ are evidence support values of $m_1, m_2, m_3, \ldots, m_n$, the credit value $Cred_i$ of m_i is as follows

$$Cred_i = \frac{Sup_i}{\sum_{j=1}^{n} Sup_j} \tag{9}$$

Evidence credit is the weight of evidence in traditional conflict management. Referring to Example 2, similarity of evidence may collide, and conflict management only based on similarity will be invalid. To overcome the shortage, conflict management based on similarity and a matrix named BPA-Matrix which depicts the BPA sequence in BOE is proposed.

The method proposed

According to Example 2, conflict management only based on similarity of evidence is not sufficient. And, BPA sequence needs to be computed in conflict management to diminish the collision of similarity. The frame of conflict management proposed in this article is shown in Figure 1.

There are three mainly parts in our method: BPA-Factor calculation, evidence credit calculation, and the combination of them. In the beginning of BPA-Factor calculation part, BPA sequence in BOE is converted into a matrix named BPA-Matrix, and the difference of each BPA-Matrix is transferred into a BPA-Factor

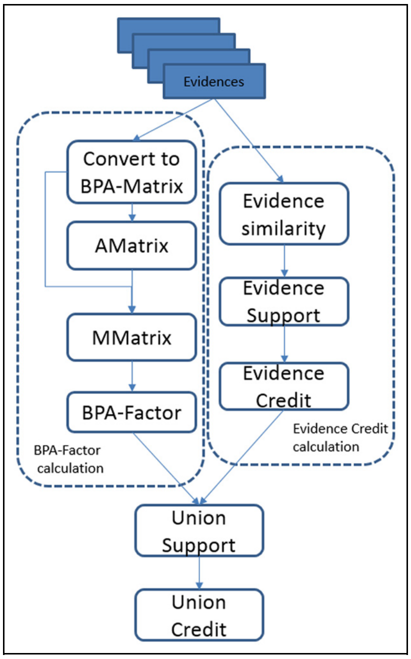

Figure 1. Frame of the proposed conflict management method.

which is marked as F in the end of this part. Evidence credit calculation part acts as the traditional conflict management, and the evidence credit based on similarity is computed. In the last part, union support is defined as the fusion of BPA-Factor and evidence credit, and union credit realizes the determination of each evidence weight based on union support.

BPA-Factor calculation

Assuming that D_1 and D_2 are two matrixes, the equation $D_3 = D_1 \times D_2$ can be considered as the amplifier and exchange on columns of D_1 or lines of D_2. The evidence can be considered as a single line matrix D_1, and the BOE with BPA sorted by the size of each BPA can be considered as another single line matrix D_3. With D_1 and D_3 being settled, D_2 can be obtained by Definition 5.

Definition 5 (BPA-Matrix of evidence). BPA-Matrix is a matrix that BPA sequence can be sorted by multiplying it. BPA-Matrix is marked as *BMatrix* in this article. The dimension of *BMatrix* is $2^{\Omega} \times 2^{\Omega}$ where Ω is the frame of discernment. The *BMatrix* of evidence m can be obtained by the following steps:

1. Set a new array R that contains all subsets of $P(\Omega)$, the array belongs to all evidence and it is generated only once;
2. Expand the BOE of m by setting BPA to 0 which exists in R, but not in m, and the expanded BOE is marked as v;
3. To each BPA in v, mark its index in R as i_{BPA}, just as $R[i_{BPA}] = \text{BPA}$;
4. A new array t is formed by sorting all BPAs in v from big to small and mark the index of each BPA in t as j_{BPA};
5. Create a matrix *BMatrix* with the dimension $2^{\Omega} \times 2^{\Omega}$. For each BPA in v, set $BMatrix_{i_{BPA}, j_{BPA}} = 1$ and the other elements in *BMatrix* as 0.

A brief example is given in the following section to illustrate the calculation process of BPA-Matrix.

Example 3 (BPA-Matrix calculation). Assuming that m is a piece of evidence under the frame of discern Ω: $\{A,B,C\}$, and the BOE of m is as follows

$$m : m(A) = 0.3,\ m(B) = 0.2,\ m(AC) = 0.5$$

First, we need to specify an array R that contains all subsets of Ω

$$R = (A, B, C, AB, AC, BC, ABC, \varnothing)$$

Second, v is obtained by expanding m

$$\begin{aligned} &m : m(A) = 0.3,\ m(B) = 0.2,\ m(AC) = 0.5 \Rightarrow \\ &v : v(A) = 0.3,\ v(B) = 0.2,\ v(C) = 0, v(AB) = 0, \\ &v(AC) = 0.5,\ v(BC) = 0,\ v(ABC) = 0,\ v = (\varnothing) = 0 \end{aligned}$$

To each BPA in v, mark its index in R as i_{BPA}

$$\begin{gathered} i_A = 1,\ i_B = 2,\ i_C = 3,\ i_{AB} = 4,\ i_{AC} = 5,\ i_{BC} = 6, \\ i_{ABC} = 7,\ i_{\varnothing} = 8 \end{gathered}$$

A new array t can be obtained by sorting each BPA in v from big to small

$$\begin{gathered} t = (t(AC) = 0.5,\ t(A) = 0.3,\ t(B) = 0.2,\ t(C) = 0, \\ t(AB) = 0,\ t(BC) = 0,\ t(ABC) = 0,\ t(\varnothing) = 0) \end{gathered}$$

Here, we get a new index j_{BPA} of each BPA in t

$$\begin{gathered} j_{AC} = 1,\ j_A = 2,\ j_B = 3,\ j_C = 4,\ j_{AB} = 5,\ j_{BC} = 6, \\ j_{ABC} = 7,\ j_{\varnothing} = 8 \end{gathered}$$

To each BPA in v, set $BMatrix_{i_{BPA}, j_{BPA}} = 1$, and the other elements in *BMatrix* $= 0$

$$BMatrix = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

BMatrix is the BPA-Matrix of to m. And, we can find BPAs in BOE can be sorted by multiplying the BPA-Matrix of it, just as the equation below

$$(0.3, 0.2, 0, 0, 0.5, 0, 0, 0) \times \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = (0.5, 0.3, 0.2, 0, 0, 0, 0, 0)$$

Each evidence is mapped into a *BMatrix* by Definition 5, and a new matrix *AMatrix* can be obtained as follows

$$AMatrix = \frac{1}{n}\sum_{i=1}^{n} BMatrix_i \tag{10}$$

n is the number of evidence, *AMatrix* is not only an overview of all *BMatrix* but also a standard in computing the difference of each *BMatrix*. The difference can be marked as *MMatrix* which is defined as follows

$$MMatrix_i = BMatrix_i - AMatrix \tag{11}$$

Each evidence is mapped into a *BMatrix* and a *MMatrix*. However, *MMatrix* is a matrix and it is inconvenient to be combined with other parameters. To overcome the shortage, BPA-Factor marked as F is introduced to realize the process that transforming *MMatrix* into a numerical value which is defined in the following section.

Definition 6 (BPA-Factor of evidence). Assume that $m_1, m_2, m_3, \ldots, m_n$ is a set of evidence under a same frame of discernment. And, $MMatrix_1, MMatrix_2, MMatrix_3, \ldots, MMatrix_n$ are *MMatrixes* belonging to $m_1, m_2, m_3, \ldots, m_n$. The BPA-Factor F_i belonging to m_i is as follows

$$F_i = \frac{n \times e^{-\|MMatrix_i\|}}{\sum_{j=1}^{n} e^{-\|MMatrix_j\|}} \tag{12}$$

$\|MMatrix_i\|$ is the norm of $MMatrix_i$, and the larger $\|MMatrix_i\|$ is, the bigger the difference of two BPA sequences is. Note that the value range of BPA-Factor is not [0, 1]. Based on the previous steps, difference of each BPA sequence is represented as a numerical value F, and the following operation is similarity calculation which is shown in section "Evidence credit calculation based on similarity."

Evidence credit calculation based on similarity

According to section "Similarity calculation," similarity calculation of evidence is realized by two kinds of methods. The first kind is based on computing the difference between two pieces of evidence directly, and the second is based on evidence distance. The similarity calculation used in this article is the method based on evidence distance which is marked as $sim(m_i, m_j)$, and we can find $sim(m_i, m_j) = sim_{Jou}(m_i, m_j)$ easily.

Based on equation (8), evidence support can be obtained as follows

$$Sup_i = \sum_{j=1, j\neq i}^{n} sim(m_i, m_j)$$

With evidence support is obtained, evidence credit can be obtained based on equation (9)

$$Cred_i = \frac{Sup_i}{\sum_{j=1}^{n} Sup_j}$$

Combining BPA-Factor and evidence credit

According to Example 2 in section "Preliminaries," conflict management only based on similarity is not sufficient, and BPA-Matrix is introduced to diminish the collision of similarity. To realize the combination of BPA-Matrix calculation and similarity calculation, a new parameter named union support is introduced. The definition of union support is described in the following section.

Definition 7 (union support of evidence). Assuming that $m_1, m_2, m_3, \ldots, m_n$ is a set of evidence under a same frame of discernment, $Cred_1, Cred_2, Cred_3, \ldots, Cred_n$ and $F_1, F_2, F_3, \ldots, F_n$ are evidence credits and BPA-Factors of $m_1, m_2, m_3, \ldots, m_n$, the union support $USup_i$ belonging to m_i is as follows

$$USup_i = F_i \times Cred_i \tag{13}$$

Note that the value range of union support is not [0, 1]. To realize the weight determination of each evidence, another process is needed to transform union support into a value with value range [0, 1]. The value is marked as union credit and is defined in the following section.

Definition 8 (union credit of evidence). Assuming that $m_1, m_2, m_3, \ldots, m_n$ is a set of evidence under the same frame of discernment, and $USup_1, USup_2, USup_3, \ldots, USup_n$ are union support values belonging to $m_1, m_2, m_3, \ldots, m_n$, and the union credit $UCred_i$ belonging to m_i is as follows

$$UCred_i = \frac{USup_i}{\sum_{j=1}^{n} USup_j} \tag{14}$$

Union credit is the weight of evidence in conflict management. Compared with previous conflict management only based on similarity, conflict management in this article is sensitive to BPA sequence in BOE. The exchange of BPA sequence will lead to the alternation of union credit and similarity collision is diminished. In addition, equation (14) can be expressed as equation (15) using primary similarity and BPA-Matrix

$$UCred_i = \frac{\dfrac{n \times e^{-\left\| BMatrix_i - \frac{1}{n}\sum_{t=1}^{n} BMatrix_t \right\|}}{\sum_{k=1}^{n} e^{-\left\| BMatrix_k - \frac{1}{n}\sum_{t=1}^{n} BMatrix_t \right\|}} \times \dfrac{\sum_{x=1, x\neq i}^{n} sim(m_i, m_x)}{\sum_{j=1}^{n}\sum_{y=1, y\neq j}^{n} sim(m_j, m_y)}}{\sum_{v=1}^{n}\left(\dfrac{n \times e^{-\left\| BMatrix_v - \frac{1}{n}\sum_{t=1}^{n} BMatrix_t \right\|}}{\sum_{u=1}^{n} e^{-\left\| BMatrix_u - \frac{1}{n}\sum_{t=1}^{n} BMatrix_t \right\|}} \times \dfrac{\sum_{x=1, x\neq v}^{n} sim(m_v, m_x)}{\sum_{j=1}^{n}\sum_{y=1, y\neq j}^{n} sim(m_j, m_y)} \right)} \tag{15}$$

Experiments

In this section, two experiments are given to compare the effect of the four representative methods for conflict management.

Experiment 1

Suggest that m_1, m_2, m_3, m_4, m_5 are four D-S evidence under a same frame of discernment $\Omega : \{A, B, C\}$ and the BOE of each evidence is given as follows

$$m_1 : m_1(A) = 0.7, m_1(B) = 0.2, m_1(C) = 0.1$$
$$m_2 : m_2(A) = 0.1, m_2(B) = 0.2, m_2(C) = 0.7$$
$$m_3 : m_3(A) = 0.4, m_3(B) = 0.2, m_3(C) = 0.4$$
$$m_4 : m_4(A) = 0.8, m_4(B) = 0.15, m_4(C) = 0.05$$

The first attribution to be compared is the similarity value in different methods which is shown below. From the similarity calculation results in Tables 1–3, it is obvious that the similarity values that m_1 and m_2 towards m_3 are same; however, m_1 is different from m_2. This phenomenon is the collision of similarity. In our method, the collision can be detected by BPA-Matrix and represented as BPA-Factor in this article. The BPA-Factor of each evidence is shown in Figure 2.

Table 1. Similarity values in proposed method.

Similarity	m_1	m_2	m_3	m_4
m_1	1	0.4	0.7	0.913397
m_2	0.4	1	0.7	0.323612
m_3	0.7	0.7	1	0.622508
m_4	0.913397	0.323612	0.622508	1

Table 2. Similarity values in Zhao et al.'s method.

Similarity	m_1	m_2	m_3	m_4
m_1	1	0.244948	0.542161	0.768622
m_2	0.244948	1	0.542161	0.187311
m_3	0.542161	0.542161	1	0.528957
m_4	0.768622	0.187311	0.528957	1

Table 3. Similarity values in Wen et al.'s method.

Similarity	m_1	m_2	m_3	m_4
m_1	1	0.333333	0.816496	0.992908
m_2	0.333333	1	0.816496	0.241969
m_3	0.816496	0.816496	1	0.756205
m_4	0.992908	0.241969	0.756205	1

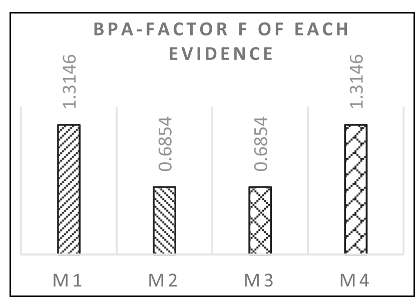

Figure 2. BPA-Factor of each evidence.

Based on similarity values and BPA-Factor above, evidence support or union support can be obtained and is shown in Table 4.

Support calculation of evidence is not the last step in conflict management, and evidence credit or union credit in this article can be obtained by evidence support. Evidence credit or union credit is shown in Table 5.

According to the tables above, the proportion of each evidence credit value in the sum of all the credit values is given in Figure 3, which is proportional to the weight of evidence in the final combination.

As m_2 or m_3 is conflicting evidence, the proportions of its credit values should be as small as possible. Oppositely, m_1 or m_4 should have larger credit proportion for it is not conflicting evidence. From the

Table 4. Comparison of evidence supports.

Method	Evidence			
	m_1	m_2	m_3	m_4
Wen et al.	2.142738	1.391799	2.389198	1.991083
Wang et al.	2.013397	1.423613	2.022508	1.859518
Zhao et al.	1.555733	0.974422	1.613279	1.484891
Method proposed	0.361647	0.133307	0.189387	0.334007

Table 5. Comparison of evidence credits.

Method	Evidence			
	m_1	m_2	m_3	m_4
Wen et al.	0.896844	0.582538	1.00000	0.833369
Wang et al.	0.275091	0.194508	0.276335	0.254066
Zhao et al.	0.276411	0.173128	0.286639	0.263825
Method proposed	0.355131	0.130905	0.185975	0.327989

experiment result, it is obvious that the proportion of evidence credit value computed by our method is the most reasonable.

Experiment 2

Example 2 depicts a more general case that evidence is gathered from the real world, and the collision of similarity may occur or not. We gathered a set of evidence from five temperate sensors as m_1, m_2, m_3, m_4, and m_5, and the BOE of each evidence is shown as follows

$$m_1 : m_1(A) = 0.4, m_1(B) = 0.15, m_1(C) = 0.45$$
$$m_2 : m_2(B) = 0.9, m_2(C) = 0.1$$
$$m_3 : m_3(A) = 0.68, m_3(B) = 0.07, m_3(AC) = 0.25$$
$$m_4 : m_4(A) = 0.45, m_4(B) = 0.1, m_4(AC) = 0.45$$
$$m_5 : m_5(A) = 0.59, m_5(B) = 0.1, m_5(C) = 0.01,$$
$$m_5(AC) = 0.3$$

The similarity values of evidence are shown in Tables 6–8.

According to three tables below collision of similarity does not occur. However, BPA sequences in BOEs are different, and the biggest focal element in each evidence may be different. This kind of difference can be represented by BPA-Factor F, and Figure 4 is the comparison of each BPA-Factor.

Referring to the giving evidence, we can find that m_1 and m_2 are conflicting evidence. BPA-Factor of them is smaller than that of m_3, m_4 and m_5. Based on BPA-Factor and similarity, evidence support of each evidence in each method can be obtained and is shown in Table 9.

Based on evidence support, evidence credit can be obtained by Definition 4. And, the comparison of each evidence credit is shown in Table 10.

Table 6. Similarity values in proposed method.

Similarity	m_1	m_2	m_3	m_4	m_5
m_1	1	0.35	0.608018	0.660884	0.648432
m_2	0. 35	1	0.172715	0.221379	0.213806
m_3	0.608018	0.172715	1	0.845404	0.938355
m_4	0.660884	0.221379	0.845404	1	0.900752
m_5	0.648432	0.213806	0.938355	0.900752	1

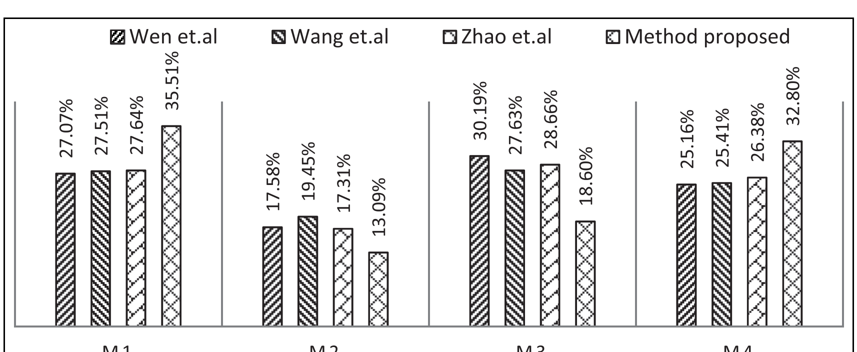

Figure 3. Comparison of proportion of evidence credit.

Table 7. Similarity values in Zhao et al.'s method.

Similarity	m_1	m_2	m_3	m_4	m_5
m_1	1	0.240134	0.420363	0.308431	0.396422
m_2	0.240134	1	0.077599	0.117835	0.116874
m_3	0.420363	0.077599	1	0.621383	0.692199
m_4	0.308431	0.117835	0.621383	1	0.625076
m_5	0.396422	0.116874	0.692199	0.625076	1

Table 8. Similarity values in Wen et al.'s method.

Similarity	m_1	m_2	m_3	m_4	m_5
m_1	1	0.320357	0.625506	0.487843	0.615070
m_2	0.320357	1	0.095582	0.154281	0.150106
m_3	0.625506	0.095582	1	0.907443	0.991596
m_4	0.487843	0.154281	0.907443	1	0.951816
m_5	0.615070	0.150106	0.991596	0.951816	1

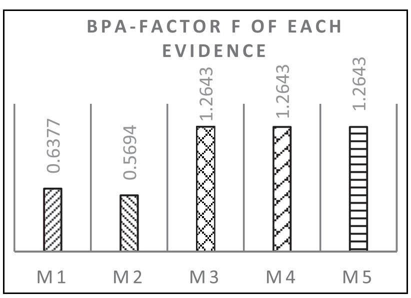

Figure 4. BPA-Factor of each evidence.

Based on the values in tables above, the proportion of each evidence credit is shown in Figure 5.

Within the giving evidence, m_1 and m_2 are conflicting evidence. The proportions of their credits are the smallest in our method. And, the proportions of m_3, m_4, m_5 computed by our method are the largest. It is obvious that the proportion of evidence credit value computed by our method is the most reasonable.

Two experiments above describe two kinds of cases that similarity collision occurs or not. As BPA-Matrix is an extra step, there should be more time cost in our proposed scheme when compared with the methods only based on similarity. When the number of evidence is n, we need to compute similarity value for n^2 times, but BPA-Matrix is computed only n times, that is, compared with the time cost of similarity calculation, the time cost of BPA-Matrix computing is tiny. As

Table 9. Comparison of evidence supports.

Method	Evidence				
	m_1	m_2	m_3	m_4	m_5
Wen et al.	2.048777	0.720326	2.620129	2.501383	2.708589
Wang et al.	2.267333	0.957901	2.564492	2.628419	2.701347
Zhao et al.	1.365350	0.552443	1.811545	1.672726	1.830572
Method proposed	0.130029	0.049049	0.291588	0.298856	0.307148

Table 10. Comparison of evidence credits.

Method	Evidence				
	m_1	m_2	m_3	m_4	m_5
Wen et al.	0.756400	0.265941	0.967341	0.923500	1.000000
Wang et al.	0.203906	0.086146	0.230630	0.236379	0.242937
Zhao et al.	0.188776	0.076382	0.250468	0.231274	0.253098
Method proposed	0.120769	0.045556	0.270823	0.277574	0.285275

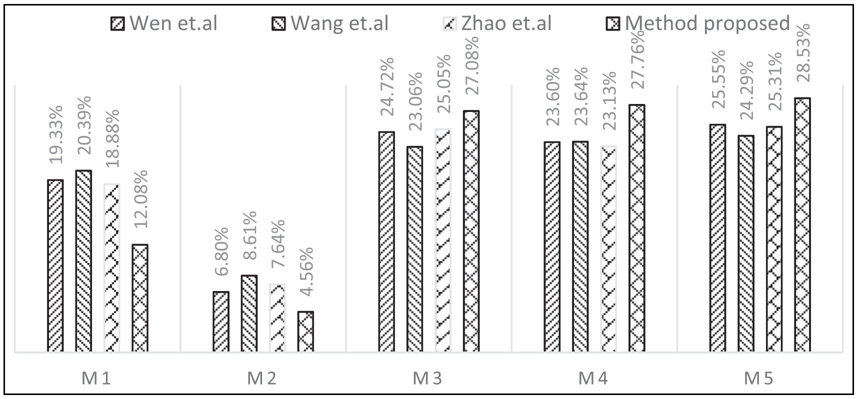

Figure 5. Comparison of proportion of evidence credit.

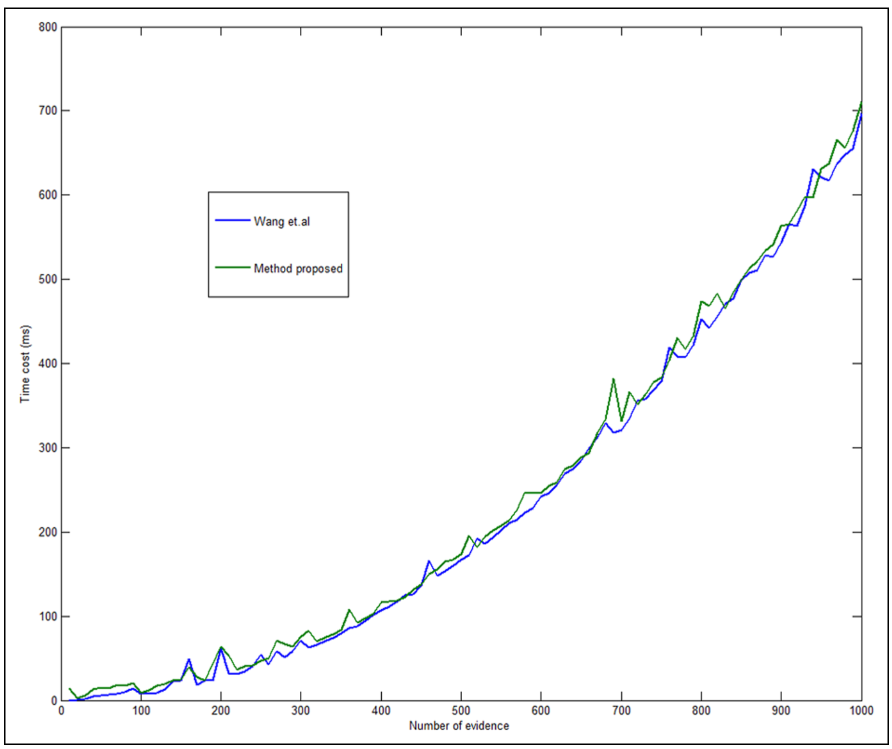

Figure 6. Time cost in two management methods.

similarity calculation method used in conflict management method proposed by Wang et al. and this article is same, a comparison of time cost in processing same evidence scale on a same platform (CPU: i7 4710, GPU: GTX960, RAM: 16G, ROM: 250G SSD) is shown in Figure 6.

From Figure 6, time costs by two methods are almost same when handling with same evidence. However, the weight determination by our conflict management method is more reasonable. Based on the comparisons above, it shows that our scheme is a better conflict management method for assigning more reasonable weight to evidence with very tiny performance loss.

Conclusion

In this article, a new conflict management method based on similarity and BPA-Matrix is proposed. Compared with the previous methods, similarity collision is diminished, and the weight determination is more reasonable. In more general cases, the weight determination by our method is more reasonable no matter whether similarity collision occurs or not. In the end, we compared the time cost of our scheme and the method of Wang et al. which is based on similarity. The experiment result shows that the time cost in the two methods is almost same; however, the proportion of evidence credit value in our scheme is more reasonable, which means that the determination of evidence weight is more reasonable. The determination of weights assigned to BPA-Matrix and similarity in conflicting management will be a new point in our future research.

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