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An improvement for combination rule in evidence theory

Jian Wang [a,*], Kuoyuan Qiao [b], Zhiyong Zhang [b]

[a] Zhengzhou University, 100, Science Avenue, Zhengzhou, Henan, 450001, China
[b] Henan University of Science & Technology, 263, Kaiyuan Avenue, Luoyang, Henan, 471023, China

HIGHLIGHTS

- Analyzing and illustrating the phenomenon and the reason of similarity collision in evidence theory.
- Introducing the Basic Probability Assignment sequence computing in the combination process to reduce the effect that similarity collision impacts on evidence weights.
- Proposing a new evidence combination rule which is attested to have the best F-score under the same dataset when compared with other methods.

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ABSTRACT

Evidence theory is an effective tool to make decision from ambiguity, which has been widely used in target recognition, decision making, optimization problem. To reduce its impact on combination results, the conflicting evidence should be assigned to a smaller weight than others when being combined. However, due to the phenomenon of similarity collision, the weight for conflicting evidence probably cannot be reduced effectively in present combination rules for similarity is the main criterion. In this paper, based on the analysis and illustration of similarity collision, a new combination rule is proposed, in which, the impact of similarity collision on evidence weights are reduced obviously by introducing the Basic Probability Assignment sorting before the final combination. In the experiment part, two sets of experiments are designed to show the superiority of the proposed method by comparing the size of each Basic Probability Assignment belonging to the correct decision and the F-Score of classification under the dataset Iris.

1. Introduction

The theory of evidence proposed by Dempster and Shafer [1] is an effective tool to make decision from uncertain information [2]. It is widely applied in many fields [3] such as decision making [4–6], reliable analyzing [7–9], relationship strength calculation [10], communication science [11,12] and optimal computing [13–15]. Traditionally, unreasonable evidence which is sent back by flawed devices is named as conflicting evidence and combination rule of evidence [16] may be invalid [17,18] when conflicting evidence exists.

To diminish the effect of conflicting evidence, Dubois [19] proposed a combination rule based on transforming the intersection parts of evidence into union parts. However, the method performs poorly when the degree of conflict is high [20]. Murphy [21] proposed a evidence combination method based on calculating the average of all evidence, but Murphy treats all evidence with same weights. To realize a better weight determination of each evidence, similarity of evidence is introduced to compute the conflict degree of each evidence [4–8].

Besides, AM (Ambiguity Measure) [22] of evidence is optional process to modify the weights of evidence. Weights of evidence with big ambiguity part should be smaller for the information it contains is less. Many scholars proposed their methods to realize the calculation of AM [23–29]. Han [30] proposed an evidence combination rule based on Ambiguity Measure method proposed by Deng [31]. Wang [32] proposed another evidence combination rule based on information entropy. Jiang [33] proposed a novel combination rule based on similarity of evidence and penalty function. Zhao [34] proposed a new combination rule based on similarity and support function k_{new}. Xiao [35,36] proposed two combination methods for evidence theory, the former one is based on evidence distance and fuzzy preference while the latter one is based on evidence similarity and Belief Function Entropy. However, both two schemes above still suffer from collision of similarity.

Even though combination rule of evidence has been improved by many scholars, the case that two different pieces of evidence share a same similarity value towards a same evidence is easy to be found, and this phenomenon is collision of similarity. It may lead to

* Corresponding author.
E-mail addresses: iejwang@zzu.edu.cn (J. Wang), qiaokuoyuan@126.com (K. Qiao), xidianzzy@126.com (Z. Zhang).
URL: http://sigdrm.org/~zzhang (Z. Zhang).

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unreasonable evidence weights and the wrong combination result following which means an wrong decision. In this paper, we found the similarity collision reason and got the method to decrease its effect on evidence combination result. Meanwhile, we presented a complete combination rule based on it, which is proved to have better performance than other compared schemes.

There are 6 parts in this paper. In the Section 1, we introduced what evidence theory and similarity collision, which triggered our research are. In the following part, related works about evidence theory are illustrated to overview the latest research about evidence theory. In the Section 3, we described some formulas and equations used in this paper. And in the "Method presentation" part, which is the Section 4, our study is unfolded in five subparts. In the Section 5, we implemented three methods which were proposed recently, the performance of the methods above and ours were compared in two sets of experiments. In the last part, we summarized our study where both a conclusion and a research direction were made.

2. Related works

In evidence theory each evidence contains several potential decisions which are named as focal-elements, and the probability that the focal-element is the correct decision is denoted as BPA (Basic Probability Assignment). Although Dempster and Shafer proposed the basic combination rule, it will be invalid when highly conflicting evidence is combined [21]. To overcome the shortage above, smaller weights are assigned to conflicting evidence based on evidence similarity. It can be realized by directly calculation between evidence and indirectly computing based on evidence distance [37]. To realize calculation of evidence distance, Cuzzolin [38] explained the distance of evidence in the view of geometric. Jousselme [39] proposed an evidence distance computing method based on different matrix, and similarity of evidence can be obtained by using 1 minus evidence distance. Wen [40] proposed another method based on the cosine value between evidence. Based on similarity of evidence, Deng [31] transferred similarity values into weight of evidence. Wang [32] improved Murphy's [21] combination rule by modifying weights of evidence. Wang [41] proposed a novel method for determined similarity collision but no combination rule is proposed.

Besides, in the process of evidence combination, the greater the ambiguity degree of the evidence is, the smaller its weight should be. Dubois [23] proposed an ambiguity measure method based on computing the difference of each BPA in BOE (Body of Evidence). Yager [24] proposed an ambiguity measure method based on computing BPAs and *Pl* (*Pl* is introduced in Section 3). Kilr [25] proposed another ambiguity measure method based on replacing *Pl* by the intersection of each BPA. George [26] improved the computing speed by simplifying exponential algorithm in Kilrs method. Kilr [28] proposed another method based on computing the distribution of each BOE but the computation process is difficult.

3. Preliminaries

3.1. Basic evidence combination rule

Let θ be a frame contains N distinct elements $\{H_1, H_2, H_3, \ldots, H_N\}$. Each element H_i is exclusive and exhaustive to the others, A is a subset of $P(\theta)$ which $P(\theta) = 2^{\theta}$. $m(A)$ is a function that maps A to [0,1] and satisfies the following conditions:

$$m(\emptyset) = 0; \qquad \sum_{A\subseteq\theta} m(A) = 1$$

Based on m and frame θ, Pl is another function which satisfies the following conditions:

$$Pl(A) = \sum_{B\cap A\neq\emptyset} m(B) = 1 - \bar{A}; \qquad m(A) \leq Pl(A)$$

$\bar{A}$ represents the complement set of A. Based on $m(A)$ and $Pl(A)$, $[m(A), Pl(A)]$ depicts the probability scope that A may be true. Noting that, all the BPAs that constitute BOE should be positive. All the BPAs of focal-elements will constitute the body of evidence (BOE) as below:

$$m : m(A), m(B), m(C), \ldots, m(AB), \ldots, m(\theta) \tag{3.1}$$

In (3.1), $m(A)$ or $m(B)$ are mass functions which represent the BPA of A or B. And a set of evidence can be combined based on the combination rule proposed by Dempster:

$$m(A) = \begin{cases} 0 & A = \emptyset \\ \frac{\sum_{A_i\cap A_j=A} m_1(A_i)m_2(A_j)}{C} & A \neq \emptyset \end{cases} \tag{3.2}$$

$$C = 1 - \sum_{A_i\cap A_j=\emptyset} m_1(A_i)m_2(A_j) \tag{3.3}$$

m is the combination result of m_1 and m_2. When the number of evidence is larger than two, (3.2) and (3.3) will be transformed into (3.4) and (3.5) as bellows:

$$m(A) = \begin{cases} 0 & A = \emptyset \\ \frac{\sum_{A_{i1}\cap A_{j2}\cap,\ldots,\cap A_{in}=A} m_1(A_{i1})m_2(A_{j2})\ldots m_n(A_{in})}{C} & A \neq \emptyset \end{cases} \tag{3.4}$$

$$C = 1 - \sum_{A_{i1}\cap A_{i2}\cap\ldots A_{in}=\emptyset} m_1(A_{i1})m_2(A_{i2})\ldots m_n(A_{in}) \tag{3.5}$$

According to (3.2)–(3.5), combination rule meets the law of commutation and the law of association [4]. Although the evidence combination can be realized by the above formula, an error combination result always follows when a piece of evidence is not supported by the others. To overcome the shortage, smaller weight is assigned to conflicting evidence, and the determination of evidence weight is mainly realized by similarity calculation and Ambiguity Measure.

3.2. Similarity of evidence

Similarity represents the degree of homogeneity among evidence, which is contrary to the distance of evidence. Based on characters of evidence distance [39], similarity of evidence should satisfy the following conditions:

1. Mapping the difference between the evidence into a value between 0 and 1 where 0 stands for a totally different, and the 1 represents a completely agreement.
2. The parameters should be unordered: $sim(m_1, m_2) = sim(m_2, m_1)$.
3. $sim(m_1, m_2) + sim(m_2, m_3)$ should be larger than $sim(m_1, m_3)$ to keep the value range compact.

Based on the conditions above, many scholars proposed their similarity calculation methods. Method proposed by Wang [32] and method proposed by Wen [40] are two methods which are most widely used. Compared with similarity calculation method proposed by Wang. Similarity method proposed by Wen is faster in calculation, but the number of elements in each focal-element is ignored. Similarity of evidence used in this paper is method proposed by Wang which is defined as:

Definition 1 (*Evidence Similarity*)**.** Assuming m_i and m_j are two BOEs under a same discernment frame. The similarity between m_1 and m_2 is:

$$sim(m_i, m_j) = 1 - d_{i,j} = 1 - \sqrt{\frac{1}{2}(\vec{m}_i - \vec{m}_j)^T D(\vec{m}_i - \vec{m}_j)} \tag{3.6}$$

$\vec{m}_i - \vec{m}_j$ is the difference between m_i and m_j in the vector form, D is a matrix which is defined as $D = \|A \cap B\| / \|A \cup B\|$. Similarity is a key attribute of evidence which is used in most combination rules. However, the phenomenon that two pairs of different evidence shares a same similarity value is easy to be found. To illustrate the case of similarity collision better, a brief example is shown:

Example 1. Assuming m_1, m_2, m_3 are three BOEs under a same discernment frame θ and the BPAs of each evidence are shown as bellows:

$m_1 : m_1(A) = 0.55, m_1(B) = 0.2, m_1(C) = 0.15, m_1(AB) = 0.1$;
$m_2 : m_2(A) = 0.15, m_2(B) = 0.2, m_2(C) = 0.55, m_2(AB) = 0.1$;
$m_3 : m_3(A) = 0.35, m_3(B) = 0.2, m_3(C) = 0.35, m_3(AB) = 0.1$;

Similarity value between m_1 and m_3 is:

$$sim(m_1, m_3) = 1 - d_{1,3} = 1 - \sqrt{\frac{1}{2}(\vec{m_1} - \vec{m_3})^T D(\vec{m_1} - \vec{m_3})} = 0.8 \tag{3.7}$$

And similarity value between m_2 and m_3 is:

$$sim(m_2, m_3) = 1 - d_{2,3} = 1 - \sqrt{\frac{1}{2}(\vec{m_2} - \vec{m_3})^T D(\vec{m_2} - \vec{m_3})} = 0.8 \tag{3.8}$$

It is obvious that $sim(m_1, m_3)$ equals $sim(m_2, m_3)$, while m_1, m_2 are not the same. This phenomenon is the collision of similarity which is referred above. The difference between m_1 and m_2 only lies on the sequence of $m(A)$ and $m(C)$ in BOE. And this kind of difference will be vanished in the plus operation just as $x+y = y+x$. In Section 4.1, we will illustrate a BPA sequence calculation method, and similarity calculation result is modified by BPA sequences.

3.3. Information entropy of evidence

As mentioned in Section 2, the amount of information that each evidence contains is different, and AM (Ambiguity Measure) is an optional process that modifies evidence weights based on the useful degree of evidence. There is a brief comparison about each Ambiguity Measure methods in [32], and the Ambiguity Measure method used in this paper is based on information entropy which is defined as bellows:

Definition 2 (*Information Entropy of Evidence*)**.** Assuming m is a piece of evidence under the discernment frame θ, the information entropy $En(m)$ of evidence m is:

$$En(m) = -\sum_{A \subseteq \theta} m(A) log_2(\frac{m(A)}{2^{\|A\|} - 1}) \tag{3.9}$$

$En(m)$ stands for the amount of information belongs to m and weight of useless evidence can be diminished based on $En(m)$.

4. Method presentation

4.1. Similarity collision and Sort-Factor

Similarity collision widely exists in present evidence similarity calculation methods for the following reason. Assuming there are two pieces of evidence m_1, m_2 and $sim(m_1, m_2) = k$. We denote each elements in m_1, m_2 as x_i and y_i; $(i \in 2^\theta)$ respectively. And the similarity equation will be: $sim(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) = k$. We treat x_i as known number, and the collision will occur when the solution of the equation above is not unique. It is apparently that the number of solution is greater than 1, which denotes similarity collision cannot be eliminated. Even though it is hard to eliminate the collision of similarity, it can be reduced by computing the sequence of each BPA in BOE. Assuming similarity between m_1 and m_2 equals k, it is apparently that $D_{x_1,y_1}(x_1-y_1)^2 + D_{x_2,y_2}(x_2-y_2)^2 + \cdots + D_{x_n,y_n}(x_n-y_n)^2 = 2 \cdot (1-k)^2$. We discuss the case that $n = 2$ and deduce the cases that n is greater than 2.

When $n = 2$, $sim(m_1, m_2) = D_{x_1,y_1}(x_1-y_1)^2 + D_{x_2,y_2}(x_2-y_2)^2 = k$. If we define $S = \frac{k}{D_{x_1,y_1} \times D_{x_2,y_2}}$, we can get $\frac{(x_1-y_1)^2}{D_{x_2,y_2}} + \frac{(x_2-y_2)^2}{D_{x_1,y_1}} = S = \frac{(y_1-x_1)^2}{D_{x_2,y_2}} + \frac{(y_2-x_2)^2}{D_{x_1,y_1}}$. The equation denotes an oval in geometry which is shown in Fig. 1.

In Fig. 1, point $y(y_1, y_2)$ denotes the value of m_2, to each point $y'(y'_1, y'_2)$ on the oval, equation $\frac{(y'_1-x_1)^2}{D_{x_1,y'_1}} + \frac{(y'_2-x_2)^2}{D_{x_2,y'_2}} = k$ will be established and similarity collision will occur. If we can limit the relationship between y_1 and y_2, such as $y_1 < y_2$, only the values on dotted line may lead to similarity collision.

When n is greater than or equals to 3, the oval is transformed into a high dimension ellipsoid which is divided into several parts $R\{r_1, r_2, \ldots r_q\}$ based on the given relationship. And similarity collision is reduced for there is only one part available in R.

Though we cannot appoint this kind of limit directly, the most reliable size relation can be deduced from the given BOEs. And the first step is to describe the size relationship of element in BOE. In mathematics, BOE can be treated as a matrix M_{BOE} with single row. If a constant matrix M_c has been appointed, the sequence of each element in BOE can be denoted by finding the matrix M_s where $M_c = M_s \times M_{BOE}$. We define M_s as the Sort-Matrix of m, which can be obtained based on Algorithm .

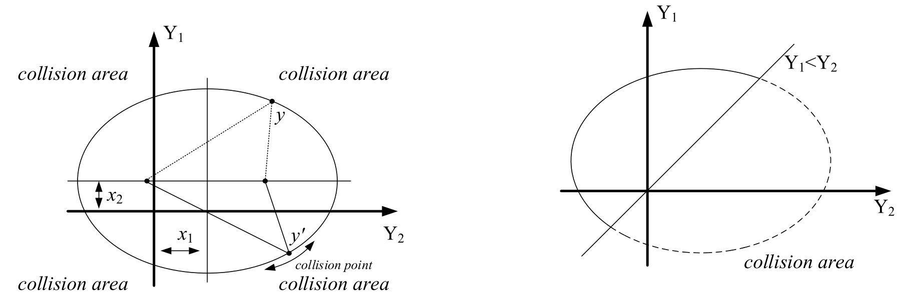
Algorithm 1: Generating Sort-Matrix of Evidence.

Input: evidence m
Output: Sort-Matrix of m

```
for each focal-element f ∈ m satisfies the condition |f| > 1 do
    set m(f) = (1/|f|) · m(f)
endfor
∀f ∈ m, set m(f) = m(f)/sum where sum = Σ_{f∈m} m(f)
specify a final sequence sq : {sf_1, sf_2, ..., sf_n} where sf_i ∈ P = 2^θ
set an empty Matrix with dimension 2^θ × 2^θ
set an ordered array A
for f ∈ P do
    set the ith element in A as m(f)
endfor
for i = 1 to size(P) do
    if i ≤ size(m) then
        find the biggest element m(f) ∈ m
        set m(f) = 0
        set j as the index of f in A
        set M_{j,i} = 1
    else
        set M_{i,i} = 1
endfor
set the rest elements of M as 0
return M as the Sort-Matrix of m
```

In Algorithm , we divide the focal-elements of evidence m into two parts. All the items in the former part are multi-elements focal-elements, and the latter part is constituted by single-elements focal-elements. After that, we reduce the effect of multi-element focal-elements on Sort-Matrix by modifying the value of each focal-element. Then, we quantify the sequence of each element by comparing the changes in the position of the focal-elements after the modified evidence is sorted. And Fig. 2 is the flow-chat of Algorithm

To illustrate the calculation process of Sort-Matrix, a brief example is given as bellows:



(a) Similarity collision area before limiting $Y_1<Y_2$ (b) Similarity collision area after limiting $Y_1<Y_2$

Fig. 1. Similarity collision oval.

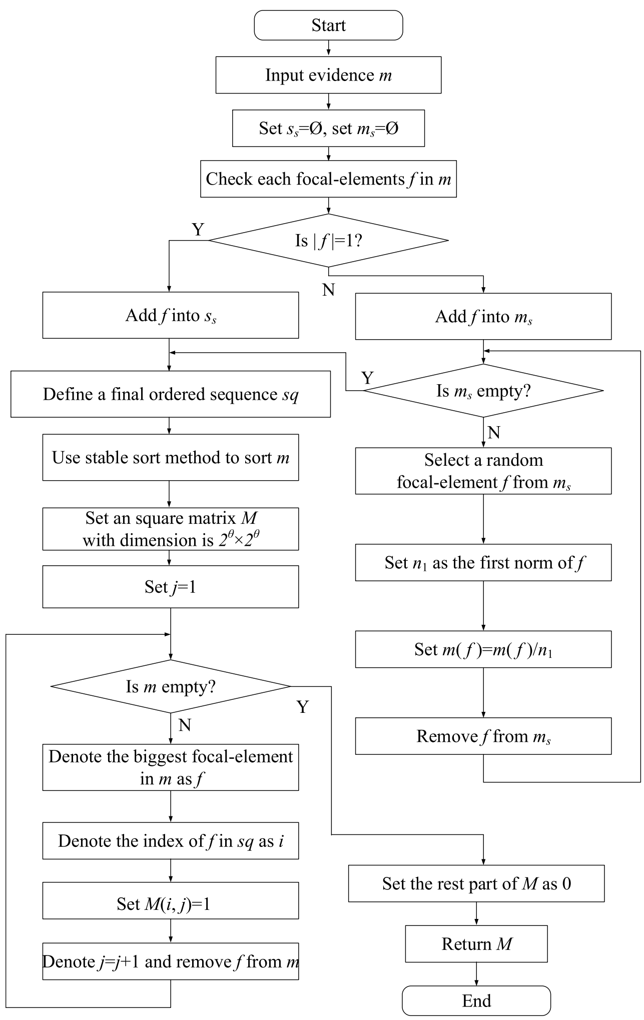


Fig. 2. Flow chat for generating Sort-Matrix.

Assuming m is a piece of evidence under the discern frame $\theta = \{A, B, C\}$, and the BOE of m is $m : \{m(A) = 0.4, m(B) = 0.1, m(AC) = 0.5\}$. According to Algorithm , $|AC| = 2 > 1$, and $m(A) = 0.533, m(B) = 0.133, m(C) = 0.334$ after the normalization (step 4). Based on m and P, $A = [0.533, 0.133, 0, 0, 0.334, 0, 0]$. As there are three elements A, B, AC in m, Sort-Matrix will be determined after three rounds of modification.
The first round: $f = A; i = 1; j = 1; M_{1,1} = 1; m = \{m(A) = 0, m(B) = 0.133, m(AC) = 0.334\}$
The second round: $f = B; i = 3; j = 2; M_{3,2} = 1; m = \{m(A) = 0, m(B) = 0.133, m(AC) = 0\}$
The last round: $f = AC; i = 5; j = 2; M_{5,2} = 1; m = \{m(A) = 0, m(B) = 0, m(AC) = 0\}$
And the Sort-Matrix of m is:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4.1}$$

Based on the Sort-Matrix of all evidence, we define the average of all Sort-Matrix as M_{avg}. The BPA sequence that M_{avg} stands for will be the most reliable sequence which mentioned in Section 4.1. Based on the difference of Sort-Matrix and M_{avg}, Deviation-Matrix of m_i can be obtained by $DM_i = M_i - M_{avg}$. DM_i is the reliable degree of BPA sequence in m_i. And the following process is to transform Deviation-Matrix into a numerical value named as Sort-Factor F.

Definition 3 (*Sort-Factor of Evidence*)**.** Assuming $m_1, m_2, m_3, \ldots, m_n$ are BOEs under the discernment frame θ, and $DM_1, DM_2, DM_3, \ldots, DM_n$ are Deviation-Matrix of $m_1, m_2, m_3, \ldots, m_n$. Sort-Factor F_i belongs to m_i is:

$$F_i = \frac{n \times e^{-\|DM_i\|}}{\sum_{j=1}^{n} e^{-\|DM_j\|}} \tag{4.2}$$

Noting that F_i may be greater than 1, but the summation of F equals n. The following step is to modify evidence support based on Sort-Factor.

4.2. Evidence support based on similarity

Definition 4 (*Evidence Support*)**.** Assuming $m_1, m_2, m_3, \ldots, m_n$ are BOEs under the discernment frame θ, $sim(m_1, m_i)$, $sim(m_2, m_i)$, $sim(m_3, m_i)$, …, $sim(m_j, m_i)$ are similarity among them, evidence support Sup_i of m_i is:

$$Sup_i = \sum_{j=1, j\neq i}^{n} sim(m_i, m_j) \tag{4.3}$$

Evidence support is the overall of evidence similarity, but similarity calculation may be invalid when collision of similarity occurs. Based on evidence support and Sort-Factor, Union Credit is defined as bellows

4.3. Union credit

Definition 5 (*Union Credit of Evidence*)**.** Assuming $m_1, m_2, m_3, \ldots, m_n$ are BOEs under the discernment frame θ, Union Credit $UCredit_i$

of m_i is:

$$UCredit_i = \frac{F_i \times Sup_i}{\sum_{j=1}^{n} F_j \times Sup_j} \tag{4.4}$$

Union Credit is the conflict evaluation based on similarity calculation and BPA sequence. But Union Credit calculation is not sufficient to determine evidence weights without useful evaluation of evidence.

4.4. General credit

In Section 4.3, Union Credit of evidence is generated, but it cannot be the evidence weight for the weight of useless evidence should be small. Based on description in Section 2, information entropy is computed to realize the useful evaluation of each evidence. General Credit is the combination of information entropy and Union Credit which is defined in Definitions 6 and 7

Definition 6 (*General Support of Evidence*)**.** Assuming $m_1, m_2, m_3, \ldots, m_n$ are BOEs under the discernment frame θ. En_i is the information entropy of m_i, and $UCred_i$ is the union credit of evidence m_i. The general support of m_i is:

$$GSup_i = UCredit_i \times \left(\frac{En_i}{\sum_{k=1}^{n} En_k}\right)^{\triangle UCredit_i} \tag{4.5}$$

$$\triangle UCredit_i = \left(\frac{1}{n}\sum_{j=1}^{n} UCredit_j\right) - UCredit_i \tag{4.6}$$

The last process to realize the weight determination is to transfer General Support into a value between 0 and 1. This value is named as General Credit which is defined as bellows:

Definition 7 (*General Credit of Evidence*)**.** Assuming evidence $m_1, m_2, m_3, \ldots, m_n$ are BOEs under the discernment frame θ, $GSup_i$ is the general support of m_i, the General Credit $GCredit_i$ of m_i is:

$$GCredit_i = \frac{GSup_i}{\sum_{j=1}^{n} GSup_i} \tag{4.7}$$

General Credit is the weight of each evidence, and combination result can be obtained based on evidence weights and their BOEs.

4.5. Combination result

As any evidence cannot be conflicted to itself, we define evidence m_{all} as the description of evidence weights and all BOEs. Each BPA in m_{all} equals $m_{all}(A) = \sum_{i=1}^{n} m_i(A) \times w_i$. w_i is the weight of evidence m_i which is also the General Credit in (4.7). However, the difference between each BPA in m_{all} is tiny. Algorithm Section 4.5 is the process that transforming m_{all} to the final combination result, and Fig. 3 is for Algorithm Section 4.5.

5. Experiment

5.1. Experiment 1

In the first experiment, we compare the size of each BPA in the combination result under a set of BOEs. We implement methods [32–34] and the data is from reference [34] which is shown in Table 1.

It is already known that the real target is θ_3, and the greater $m(\theta_3)$ in the combination result is, the better combination rule is. The size of θ_3 in different combination results determined by different combination methods are shown in Fig. 6. Refer to Fig. 6, it is apparently that $m(\theta_3)$ determined by our scheme is

Algorithm 2: Combination.

Input: evidence $m_1, m_2, \ldots, m_n$
Output: Combination result

1: compute m_{all} based on $m_1, m_2, \ldots, m_n$
2: set $BPA_{max} = 0$
3: set $Focal_{max} = \emptyset$
4: **for** $f \in 2^{\theta}$ **do**
5:     **if** $m_{all}(f) > BPA_{max}$ and $|f| = 1$ **then**
6:         set $BPA_{max} = m_{all}(f)$
7:         set $Focal_{max} = f$
8:     **end if**
9: **end for**
10: set $m = m_{all}$
11: set $k = 1 - BPA_{max}$
12: **for** $f \in 2^{\theta}$ **and** $f \neq Focal_{max}$ **do**
13:     set $m(f) = m(f) \times k$
14: **end for**
15: set sum=$\sum_{f \in 2^{\theta}} m(f)$
16: $\forall f \in 2^{\theta}$, set $m(f) = \frac{m(f)}{sum}$
17: repeat step 12-16 for n times
18: return m as the combination result of $m_1, m_2, \ldots, m_n$

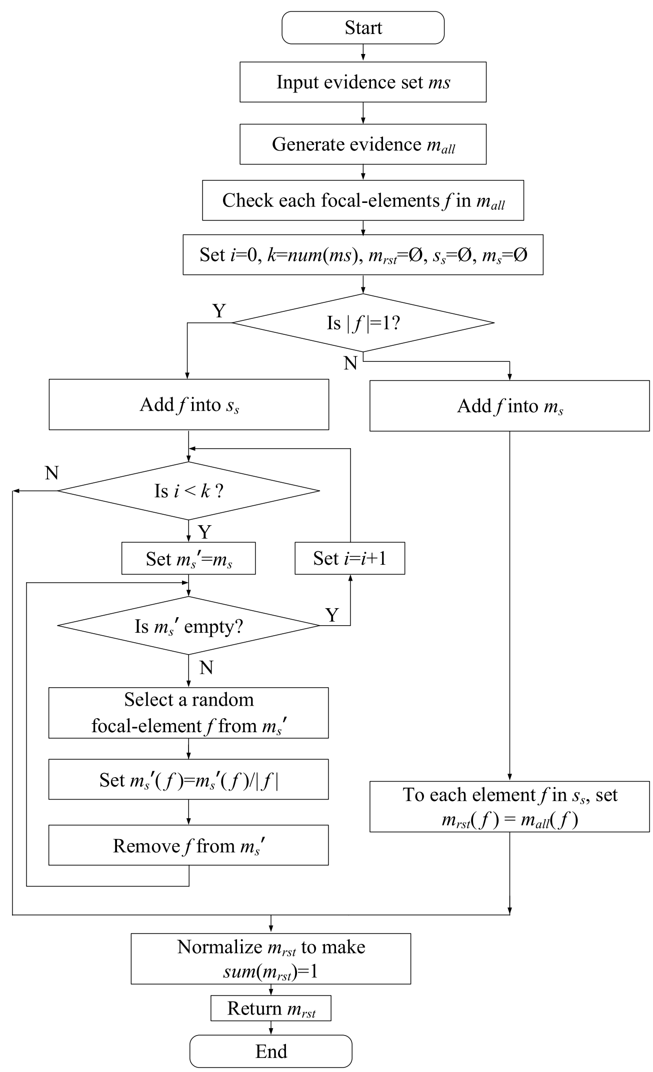


Fig. 3. Flow chart for generating combination result.

Table 1
BOEs of evidences.

	$\{\theta_1\}$	$\{\theta_2\}$	$\{\theta_3\}$	$\{\theta_1,\theta_2\}$	$\{\theta_1,\theta_3\}$	$\{\theta_2,\theta_3\}$	$\{\theta_1,\theta_2,\theta_3\}$
m_1	0.1920	0.2446	0.2634	0.1503	0.0381	0.0410	0.0707
m_2	0.2317	0.0701	0.2244	0.0671	0.2561	0.0965	0.0542
m_3	0.0686	0.1684	0.1294	0.0961	0.2271	0.1600	0.1503
m_4	0.2454	0.0765	0.2026	0.2016	0.1018	0.1519	0.0203
m_5	0.0156	0.1531	0.2248	0.2695	0.0375	0.1641	0.1354
m_6	0.0052	0.1459	0.0702	0.3437	0.1347	0.2287	0.0717
m_7	0.1725	0.0753	0.1874	0.1974	0.2143	0.1291	0.0240
m_8	0.0613	0.2447	0.0408	0.2212	0.1442	0.2668	0.0209
m_9	0.1113	0.0268	0.2419	0.0012	0.1949	0.2055	0.2184
m_{10}	0.0275	0.1303	0.0847	0.2608	0.1406	0.2968	0.0593
m_{11}	0.0981	0.0541	0.0506	0.3233	0.2156	0.2045	0.0539
m_{12}	0.2790	0.2035	0.1148	0.1679	0.1314	0.0249	0.0785
m_{13}	0.0431	0.0643	0.0839	0.1458	0.0174	0.3155	0.3302
m_{14}	0.1411	0.1406	0.0971	0.2587	0.1061	0.0320	0.2243
m_{15}	0.1233	0.0764	0.1277	0.0305	0.0417	0.2979	0.3024
m_{16}	0.2736	0.0284	0.1117	0.1680	0.3906	0.0073	0.0205
m_{17}	0.0484	0.1859	0.2096	0.1855	0.1291	0.1567	0.0849
m_{18}	0.2081	0.0528	0.1919	0.0513	0.1030	0.1748	0.2180
m_{19}	0.0234	0.2685	0.2241	0.1406	0.1259	0.1291	0.0885
m_{20}	0.1139	0.1144	0.1831	0.1780	0.1443	0.0848	0.1817
m_{21}	0.1195	0.0787	0.2106	0.1965	0.1234	0.1396	0.1317
m_{22}	0.0839	0.1217	0.1902	0.0931	0.3411	0.0787	0.0913
m_{23}	0.0636	0.0848	0.1624	0.1159	0.3441	0.1603	0.0689
m_{24}	0.2448	0.2651	0.1187	0.0301	0.0698	0.1106	0.1609
m_{25}	0.1036	0.2382	0.2810	0.0876	0.0464	0.1172	0.1260

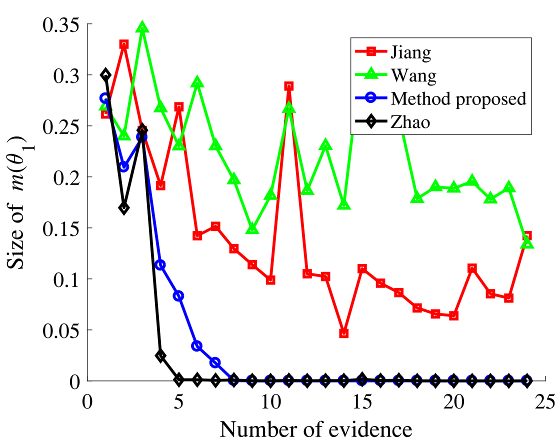


Fig. 4. Size of $m(\theta_1)$ in combination result.

the largest. Besides, the size of $m(\theta_3)$ determined by our scheme increase fastest when the number of evidence is increasing. And $m(\theta_1)$, $m(\theta_2)$, $m(\theta_1,\theta_2)$, $m(\theta_1,\theta_3)$, $m(\theta_2,\theta_3)$, $m(\theta_1,\theta_2,\theta_3)$ in different combination result are shown in Figs. 4, 5, 7, 8, 9, 10.

Refer to Figs. 4, 5, 7, 8, 9, 10, BPA of focal-elements not equal θ_3 determined by the proposed method are the smallest, which means the combination result obtained by our scheme is the best.

6. Experiment 2

In the second experiment, we compare the performance of each method under the data-set Iris which is available on http://archive.ics.uci.edu/ml/datasets/Iris. In the dataset, each item contains five values which are sepal length, sepal width, petal length, petal width and plant type. First four values are converted into four pieces of evidence [42] to predict the plant type by combining the evidence obtained. The statistics of correct decision are processed and the F-Score of different methods is compared in this part. We randomly pick up a piece of item which is given as bellows:

[6.3, 3.3, 4.7, 1.6, *Iris* − *versicolor*]

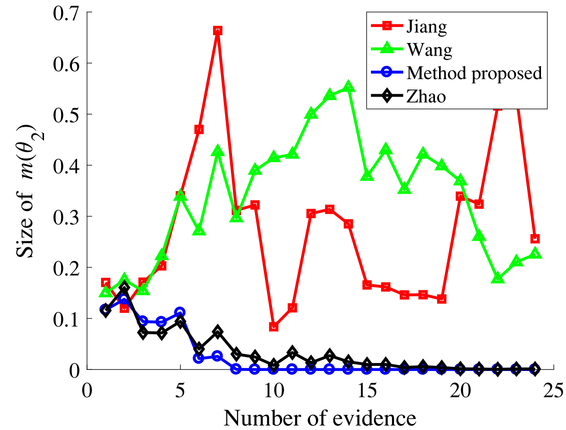


Fig. 5. Size of $m(\theta_2)$ in combination result.

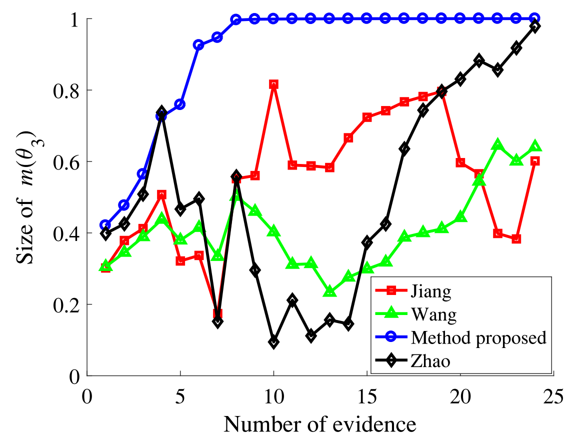


Fig. 6. Size of $m(\theta_3)$ in combination result.

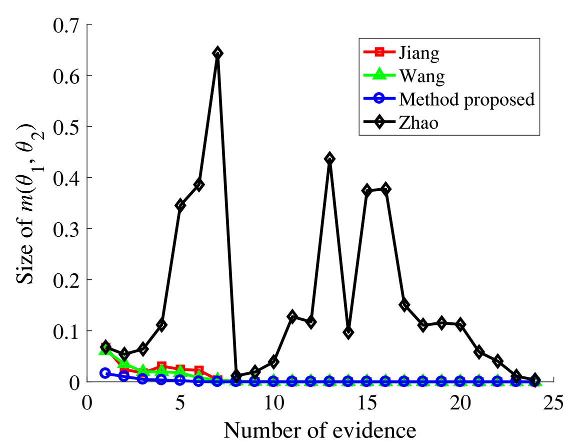


Fig. 7. Size of $m(\theta_1,\theta_2)$ in combination result.

The type of the item above is "Iris-versicolor", and the features of sepal length, sepal width, petal length and petal width is 6.3, 3.3, 4.7, 1.6 respectively. We use the method [42] to convert the values above into BOEs which are shown in Table 2.

Based on the data in Table 2, combination results obtained by several combination rules are shown in Table 3.

According to the combination results in Table 3, method proposed by Wang and method in this paper made the correct decision. We also statistic the Accuracy and Recall rate of each method when the training rate is changing (Table 4 and Fig. 11).

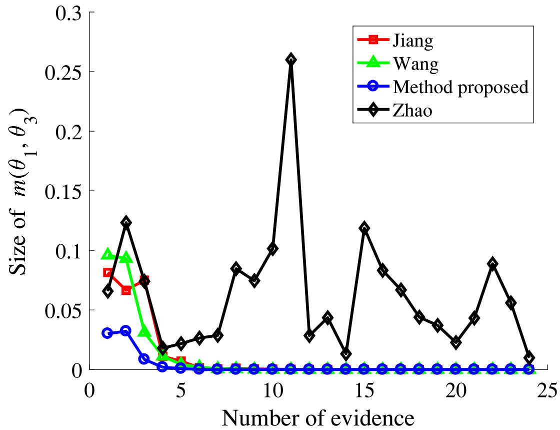


Fig. 8. Size of $m(\theta_1, \theta_3)$ in combination result.

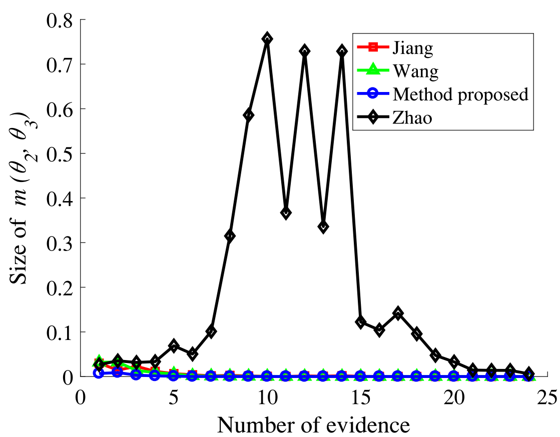


Fig. 9. Size of $m(\theta_2, \theta_3)$ in combination result.

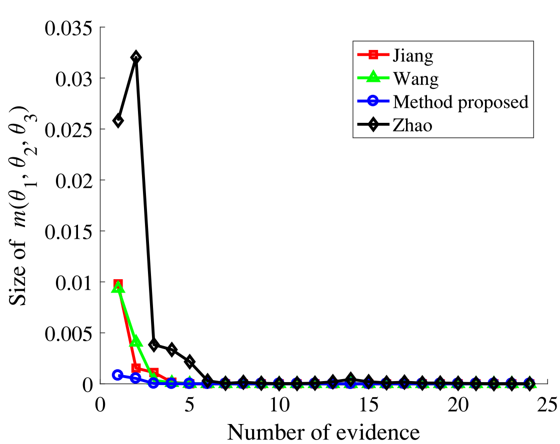


Fig. 10. Size of $m(\theta_1, \theta_2, \theta_3)$ in combination result.

From Fig. 11, we can find that the accuracy of our method is greater than other methods. And the F-Score of our scheme keeps the largest.

7. Conclusion and discussions

In this paper, a new combination rule based on similarity, BPA sequence and ambiguity measure is proposed. In our scheme, Sort-Matrix is introduced to illustrate the BPA sequence in evidence which is used to diminish the effect of similarity collision. The experiment result shows that the combination result obtained by our scheme is more suitable to make decision when compared with other methods. The implement of the proposed theory and method in some advance fields of wireless communication, such as wireless energy transfer, green communications, and cognitive radio network, will be the future work.

Discussion 1: Why not discard conflicting evidence directly?

To be honest, a larger BPA may be obtained if conflicting evidence is discarded. However, it is arbitrary to assert that conflicting evidence is wrong, especially when the number of evidence is small. Generally, conflicting evidence is assigned to smaller weight to diminish the effect that it may cause on the combination result. When the number of evidences rising, the conflicting evidence is more and more clearly and the weight of it will be tiny.

Discussion 2: Why not proposing a new similarity calculation method instead of computing BPA sequence?

Let us consider the similarity calculation process $sim(m_1, m_2) = k$ in the view of mathematics. BPAs in evidence m_1 can be considered as the known numbers in an equation and BPAs in anther evidence m_2 are considered as the unknown numbers. The number of unknown numbers is greater than 1 indicates that the solutions are not unique and the alternation of similarity calculation method is helpless to reduce similarity collision.

Discussion 3: Does the collision of similarity is eliminated?

Collision of similarity is diminished but not eliminated. Refer to Discussion 2, to diminish the collision of similarity, the solution of equation must be unique, which is not impossible. We diminished

Table 2
BOEs of evidence.

	$m_{sepallength}$	$m_{sepalwidth}$	$m_{petallength}$	$m_{petalwidth}$
$m(Setosa)$	0.07062	0.17798	0.03693	0.05014
$m(Versicolor)$	0.18406	0.08218	0.23066	0.19657
$m(Virginica)$	0.19373	0.12156	0.11875	0.10208
$m(Setosa, Versicolor)$	0.09699	0.16273	0.05673	0.07245
$m(Setosa, Virginica)$	0.10901	0.17798	0.06652	0.08382
$m(Versicolor, Virginica)$	0.20879	0.11480	0.25578	0.26745
$m(\theta)$	0.13677	0.16273	0.23458	0.22745

Table 3
Combination result obtained by different combination rules.

	Wang	Zhao	Jiang	Method proposed
$m(Setosa)$	0.05935	0.00582	0.33747	0.04257
$m(Versicolor)$	0.38024	0.17167	0.13789	0.47303
$m(Virginica)$	0.25249	0.07146	0.20776	0.08102
$m(Setosa, Versicolor)$	0.03721	0.01624	0.07583	0.05371
$m(Setosa, Virginica)$	0.04550	0.02708	0.09205	0.06115
$m(Versicolor, Virginica)$	0.17959	0.41044	0.06139	0.15289
$m(\theta)$	0.00928	0.29726	0.00647	0.13561

Table 4
Recall rate of each methods.

Rate of tanning set	Wang	Zhao	Jiang	Method proposed
0.1	0.922	0.976	0.787	0.945
0.2	0.928	0.972	0.795	0.946
0.3	0.928	0.966	0.771	0.938
0.4	0.925	0.958	0.763	0.938
0.5	0.922	0.933	0.771	0.937
0.6	0.938	0.916	0.775	0.956
0.7	0.956	1	0.807	0.979
0.8	0.969	1	0.867	1
0.9	1	1	0.805	1

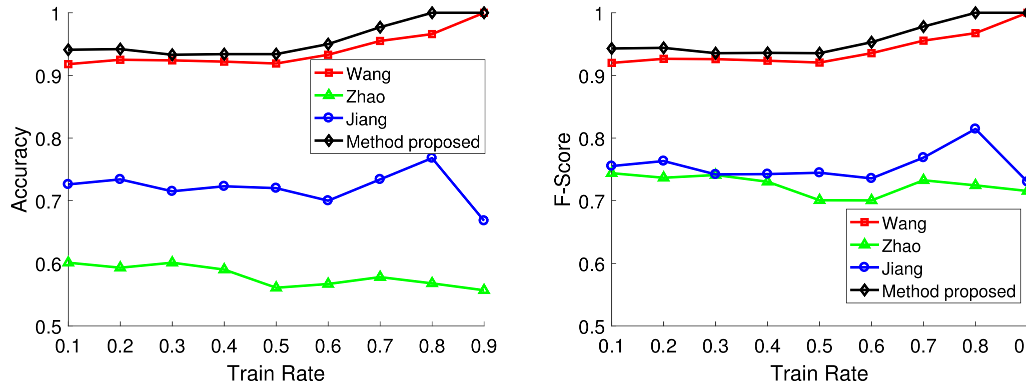


Fig. 11. Accuracy and F-Score of each method.

the collision of similarity by computing BPA sequence in BOE but we cannot eliminate it so far.

Discussion 4: What is the difference between AM and Sort-Matrix calculation?

AM and Sort-Matrix calculation are two important steps in combination. AM is the process that computing the degree of useful of the evidence, and Sort-Matrix calculation is introduced to diminish the degree of similarity collision. Evidence with large AM values may also lead to similarity collision.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.future.2018.08.010.

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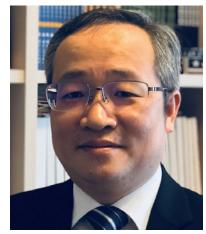
Jian Wang was born in December 1978, earned her Master, Ph.D. degrees in Computer Science from Huazhong University of Science & Technology and Tongji University, PR China, respectively. Nowadays, she is a full-time Associate Professor in College of Information Engineering, Zhengzhou University. Wang's research interests include multimedia social networks, information security, trusted computing and usage control. Recent years, she has published over 20 scientific papers in the above research fields. She is the member of the China Computer Federation (2012).

Kuoyuan Qiao was born in June 1991, earned her Master degrees in Anyang University, P.R. China. Nowadays, he is a postgraduate student in Henan University of Science & Technology.



Zhiyong Zhang born in October 1975, earned his Master, Ph.D. degrees in Computer Science from Dalian University of Technology and Xidian University, P. R. China, respectively. He was ever post-doctoral fellowship at School of Management, Xi'an Jiaotong University, China. Nowadays, he is a full-time Henan Province Distinguished Professor and Dean with Department of Computer Science, College of Information Engineering, Henan University of Science & Technology. He is also a Visiting Professor of Computer Science Department of Iowa State University. Prof. Zhang and research interests include multimedia social networks, digital rights management, trusted computing and usage control. Recent years, he has published over 80 scientific papers and edited 4 books in the above research fields, and also holds 8 authorized patents. He is IEEE Senior Member (06'M, 11'S), ACM Senior Member (08'M, 13'S), IEEE Systems, Man, Cybernetics Society Technical Committee on Soft Computing, World Federation on Soft Computing Young Researchers Committee, Membership for Digital Rights Management Technical Specialist Workgroup Attached to China National Audio, Video, Multimedia System and Device Standardization Technologies Committee. And also, he is editorial board member and associate editor of Multimedia Tools and Applications (Springer), Neural Network World, EURASIP Journal on Information Security (Springer), Social Network Analysis and Mining (Springer), Topic (DRM) Editor-inChief of International Journal of Digital Content Technology and Its Applications, leading guest editor or co-guest Editor of Applied Soft Computing (Elsevier), Computer Journal (Oxford) and Future Generation Computer Systems (Elsevier). And also, he is Chair/Co-Chair and TPC Member for numerous international conferences/workshops on digital rights management and cloud computing security.